A Quasicrystallic Domain Wall in Nonlinear Dissipative Patterns

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ABSTRACT

We propose an indirect approach to the generation of a two-dimensional quasiperiodic (QP) pattern in convection and similar nonlinear dissipative systems where a direct generation of stable uniform QP planforms is not possible. An *eightfold* QP pattern can be created as a broad transient layer between two domains filled by square cells (SC) oriented under the angle of 45 degrees relative to each other. A simplest particular type of the transient layer is considered in detail. The structure of the pattern is described in terms of a system of coupled real Ginzburg-Landau (GL) equations, which are solved by means of combined numerical and analytical methods. It is found that the transient "quasicrystallic" pattern exists exactly in a parametric region in which the uniform SC pattern is stable. In fact, the transient layer consists of two different sublayers, with a narrow additional one between them. The width of one sublayer (which locally looks like the eightfold QP pattern) is large, while the other sublayer (that seems like a pattern having a quasiperiodicity only in one spatial direction) has a width ~ 1. Similarly, a broad stripe of a *twelvefold* QP pattern can be generated as a transient region between two domains of hexagonal cells oriented at the angle of 30 degrees.

1 Introduction

Stable quasiperiodic (QP) planforms have been first discovered as equilibrium patterns in metallic alloys [1]. The simplest and most important types of the QP planforms are the ten-, eight-, and twelvefold ones (note an argument according to which only these types of the two-dimensional quasicrystals may occur in physical systems [2]). A $2n$-fold pattern is a superposition of $n \geq 4$ spatial harmonics based on a set of equal-length wave vectors $\mathbf{k}_j$ with the equal angles π/n between $\mathbf{k}_j$ and $\mathbf{k}_{j+1}$ (more general aperiodic patterns with unequal wave vector lengths and/or unequal angles between the vectors may also exist, but they have probably never been considered, except for the case of unequilateral hexagons with $n = 3$ [3]).

Then, it was predicted that the $2n$-fold structures of different particular types may exist as dynamical planforms of a stationary form in nonequilibrium systems, such as thermal convection and the like [4]. Later, the predicted pattern (a twelvefold one, in particular) was indeed experimentally observed, first, as a stable dynamical nonequilibrium planform in the Faraday ripples [5] (a large-aspect-ratio liquid layer subject to high-frequency shaking in the vertical direction), and recently in an optical cell filled with Na vapors [6]. Nevertheless, experimental generation of dynamical QP patterns is far from being straightforward; in particular, in the work [5] it was necessary to shake the liquid layer with a two-frequency quasiperiodic force at a specially selected ratio between the two frequencies. These experimental difficulties are related to the general fact that, in terms of the corresponding coupled Ginzburg-Landau (GL) equations for amplitudes of the spatial modes, a superposition of which gives rise to the pattern, a formal solution for the QP planforms always exists, but it may be stable only in a relatively narrow parametric region [4].

An objective of this work is to put forward a much easier possibility of generating eight- and twelvefold QP planforms of a finite but large size (a stripe) in "normal" systems, where direct generation of the quasicrystallic patterns does not seem feasible. To this end, we notice that the set of the wave vectors on which the eight- or twelvefold QP is based may be regarded, in an obvious way, as a superposition of two half-sets of 2 or 3 vectors, see Fig. 1 below. Each half-set, in turn, may give rise to a usual periodic pattern consisting of square cells (SC) or hexagonal cells, respectively. Thus, a natural way to generate a stripe filled with the QP pattern in a generic system (e.g., a convective layer), in which a stable QP pattern is not available, but stable periodic SC and/or hexagonal patterns do exist, is to produce it as a transient layer ("domain wall" [4]) between two large domains filled with the periodic cells, the (half-) sets of the two or three wave vectors in the two domains being oriented under the angle, respectively, 45 or 30 degrees relative to each other. The pattern in the transient layer will then be a superposition of the two periodic patterns, having a full set of the four or six wave vectors, respectively. The so generated stripe will feature a QP pattern, provided that its width is essentially larger than the wavelength (a size of the elementary cell in the corresponding periodic pattern). It is known that the latter condition can be achieved, under special but not unrealistic conditions, for domain walls in the nonequilibrium systems [4].

In fact, it may be simpler to generate, following this way, a twelvefold QP pattern between two hexagonal domains, as it is usually much easier to find a stable hexagonal structure than a stable SC one [7]. However, the theoretical analysis of the transient layer between two SC domains is much simpler, therefore in this work we concentrate on the

latter case. In any case, it should be stressed that, although stable square-cell patterns are rarer than their counterparts in the form of hexagons or rolls, examples of stable square cellular planforms are known in gas flames [8], thermal convection [9], and in some optical systems [10]. The interest to the SC planforms has been recently revived by the discovery of this pattern in a double-layer Marangoni convection (see the e-print [11] and references therein).

The rest of the paper is organized as follows. In section 2, we formulate the model. Some analytical results, which predict, in particular, that the transient layer between the two SC patterns has a complex structure, consisting of *three* layers, two broad ones and a narrow sublayer sandwiched between them, are obtained in section 3. In section 4, we display results of a direct numerical solution of the stationary real GL equations, which comply with the analytical predictions. The paper is concluded by section 5.

2 The Model

In this work, we assume a simplest configuration, shown in Fig. 1, that is going to give rise to the QP transient layer. The layer is parallel to the y axis, and the (half) sets of the two wave vectors are chosen so that in the left domain the vectors $\mathbf{k}_{1,2}$ are parallel to the x and y axes, while in the right domain both vectors $\mathbf{k}_{3,4}$ have the angle 45 degrees relative to the axes. Accordingly, the complex wave field describing the spatial distribution of physical variables is assumed to be

$$u(\mathbf{r},t)=\sum_{j=1}^{4}B_j(\mathbf{r},t)\,\exp\left(i\mathbf{k}_j\mathbf{r}\right), \tag{1}$$

where $\mathbf{r}$ is the two-dimensional coordinate, and $B_j(\mathbf{r},t)$ are slowly varying amplitude functions.

In an experiments, the chosen configuration can be created, at least in a part of the system, by means of specially selected boundary conditions, which, in turn, are imposed by the sidewalls of the experimental cell. Although we do not analyze the sidewall boundary conditions in this work, it is obvious that, to support the configuration that we consider, one will need to have a large-aspect-ratio cell with two sidewalls forming an angle 45 degrees, which is quite possible. More general configurations, with different orientations of the wave vectors relative to the layer between the two domains, can be considered similarly to what is done below, but their technical treatment will be more cumbersome.

A usual approach to the description of spatially nonuniform patterns of the domain-wall type is based on a system of coupled real Ginzburg-Landau (GL) equations for the slowly varying amplitudes, assuming that they do not depend on the coordinate y running along the stripe (wall), which is a reasonable approximation if the stripe is long enough [12, 13]. Then, as it was shown in the mentioned works, the effective diffusion coefficient (the one in front of the term $(B_j)_{xx}$) in each GL equation is

$$D_j=\left(k_j^{(x)}\right)^2, \tag{2}$$

where $k_j^{(x)}$ is the x-component of the vector. For the configuration shown in Fig. 1, the system of the real GL equations can be easily cast into a final form

$$A_t \;=\; (1/2)A_{xx}+A-A^3-2\left(g_2A^2+g_1B_1^2+g_1B_2^2\right)A, \tag{3}$$

$$(B_1)_t = (B_1)_{xx} + B_1 - B_1^3 - 2\left(2g_1A^2 + g_2B_2^2\right)B_1, \tag{4}$$

$$(B_2)_t = B_2 - B_2^3 - 2\left(2g_1A^2 + g_2B_1^2\right)B_2, \tag{5}$$

where $B_{1,2}$ are the amplitudes corresponding to the wave vectors $\mathbf{k}_{1,2}$ in the left domain (Fig. 1), and, using the obvious symmetry of the configuration shown in Fig. 1, we have set $B_3 = B_4 \equiv A$.

Note that the diffusion coefficient in Eq. (5) is zero due to Eq. (2). Generally speaking, in this case one should take into account a higher-order derivative term $\sim (B_2)_{xxxx}$ [12, 13]. However, this is not necessary while the diffusion terms do not vanish in the two other equations (see details below).

In Eqs. (3) through (5), the linear gain coefficient and the coefficient of the nonlinear self-interaction of the spatial mode are normalized to be 1, g_1 and g_2 being the coefficients of the nonlinear interaction between the modes with the angles $\alpha = 45$ and 90 degrees between their carrier wave vectors. Normally, the nonlinear interaction coefficient decreases with the increase of α, so that

$$g_2 < g_1 < 1. \tag{6}$$

The necessary and sufficient stability conditions for the SC pattern are well known [4],

$$g_2 < \frac{1}{2},\ g_1 > \frac{1}{4}\left(1 + 2g_2\right), \tag{7}$$

while the conditions providing for stability of the eightfold QP planform are

$$g_2 < \frac{1}{2},\ g_1 < \frac{1}{4}\left(1 + 2g_2\right). \tag{8}$$

An obvious feature of the two sets of the stability conditions is their incompatibility, i.e., SC and QP can never be stable simultaneously.

In fact, the inequality $g_2 < 1/2$ in the set of the conditions (7) is a cause for the relative rarity of stable SC planforms, as, despite the general property (6), the actual dependence $g(\alpha)$ is usually weak, so that g_2 is not essentially smaller than 1. Nevertheless, the full set of the SC stability conditions (7) can be satisfied in the above-mentioned physical systems.

A consequence of the conditions (8) necessary for the stability of the QP pattern is $g_1 < 1/2$, which, with regard to Eq. (6), makes the stability of the QP pattern still less feasible than that of the SC one. An objective of this work is to propose a way to produce a transient QP pattern between two *stable* SC domains with different orientations (Fig. 1) in the case when the uniform QP planform is unstable. Note that a similar approach is known as a way to generate of a broad stripe of a SC pattern between two domains of orthogonally oriented rolls in the case when the uniform SC pattern is unstable, while the rolls are stable [13].

In the case when the SC stability conditions (7) are met, it is quite reasonable to assume that the coefficients $g_{1,2}$ are close to 1/2 (because it is physically implausible to have $g_{1,2}$ much smaller than 1/2), which suggests to present them as

$$g_{1,2} \equiv \frac{1}{2}\left(1 - \mu_{1,2}\right),\ |\mu_{1,2}| \ll 1. \tag{9}$$

As it will be seen below, the smallness of $\mu_{1,2}$ naturally provides for the strip of the QP pattern to be broad, which is exactly the condition justifying the consideration of the

transient-layer patterns. Note that μ_2 must be positive according to Eq. (7), while μ_1 may formally have either sign. However, it will be shown below that the necessary solution does not exist if $\mu_1 < 0$.

Below, we will also use a parameter

$$m \equiv 2\mu_1/\mu_2, \tag{10}$$

which is, generally, ~ 1. In terms of m, the second stability condition (7) takes a very simple form, $m < 1$.

Eqs. (3), (4), and (5) must be supplemented by boundary conditions (b.c.) to guarantee that, at $x \to \pm\infty$ (recall the system is formally assumed to be infinitely large), the pattern considered asymptotically coincides with either of the two SC planforms composed of the modes B_1 and B_2 or $B_3 = B_4 \equiv A$. Obviously, this implies

$$\lim_{x\to+\infty} A(x) = 1/\sqrt{2-\mu_2} \equiv A_{\lim}, \ \lim_{x\to+\infty} B_{1,2}(x) = 0, \tag{11}$$

$$\lim_{x\to-\infty} A(x) = 0, \ \lim_{x\to-\infty} B_{1,2}(x) = A_{\lim}\,. \tag{12}$$

Formally, this set may seem overdetermined, as we add six b.c. to the fourth-order system of Eqs. (3) and (4) (Eq. (5) contains no x-derivatives). However, it is easy to check that the seemingly superfluous b.c. for B_2 are nothing else but direct corollaries of the four legitimate b.c. for A and B_1.

3 Analytical Results

A stationary version ($\partial/\partial t = 0$) of Eqs. (3), (4) and (5) can be essentially simplified, as in this case Eq. (5) becomes just an algebraic relation, that has two solutions: $B_2 = 0$, or

$$B_2^2 = 1 - (2 - m\mu_2)\, A^2 - (1 - \mu_2)\, B_1^2, \tag{13}$$

m being the parameter defined by Eq. (10). First, we consider the case when the expression (13) holds (obviously, it may hold as long as it yields $B_2^2 > 0$). Substituting it into the stationary versions of Eqs. (3) and (4), we obtain

$$\mu_2^{-1}A'' + mA - \left[\left(2\,(2m-1) - m^2\mu_2\right) A^2 + (2 - m\mu_2)\, B_1^2\right] A = 0, \tag{14}$$

$$\mu_2^{-1}B_1'' + B_1 - \left[(2-\mu_2)\, B_1^2 + (2 - m\mu_2)\, A^2\right] B_1 = 0, \tag{15}$$

the prime standing for d/dx. If, instead of Eq. (13), we take $B_2 = 0$, the stationary equations take the form

$$A'' + 2A - \left[2\,(2-\mu_2)\, A^2 + (2 - m\mu_2)\, B_1^2\right] A = 0, \tag{16}$$

$$B_1'' + B_1 - \left[B_1^2 + (2 - m\mu_2)\, A^2\right] B_1 = 0. \tag{17}$$

It is noteworthy that, although the SC patterns may be stable at $\mu_1 < 0$, i.e., $m < 0$, Eq. (14) cannot have a solution for $A(x)$ exponentially decaying at $x \to -\infty$ (see the b.c. (12)) unless $m > 0$, hence the present problem has *no solution* with $m < 0$.

Obviously, a solution to Eqs. (14) and (15) can satisfy, with regard to Eq. (13), the b.c. (12). However, the same set of equations (14) and (15) *cannot* satisfy the b.c. (11): setting $B_1 = 0$ and $A = \text{const}$, one obtains from Eq. (14)

$$A^2 = A_0^2 \equiv \frac{m}{2\,(2m-1) - m^2\mu_2}\ , \tag{18}$$

which is obviously different from the necessary limit value $A_{\text{lim}}^2 \equiv 1/\,(2-\mu_2)$. Moreover, the value of B_2^2 corresponding to A_0^2 as per Eq. (13) is different from zero, which also violates the b.c. (11).

In fact, the stationary state corresponding to the asymptotic value (18) with $B_2 \neq 0$ is another uniform pattern, which is a superposition of three spatial harmonics. This pattern is periodic in one spatial direction and quasiperiodic in the other one. As it was demonstrated in [4], this pattern is always dynamically unstable, hence a solution having it as an asymptotic state is physically irrelevant.

However, a solution to Eqs. (14) and (15) makes sense as long as it provides for $B_2^2 > 0$ according to Eq. (13). Taking the asymptotic state $A^2 = A_0^2$, $B_1^2 = 0$, one finds that it gives rise to *negative* B_2^2 exactly in the case $m < 1$, which is considered in this work, as this is the case when the SC pattern is stable, see above. Thus, Eqs. (14) and (15) should be used in the region $-\infty < x < x_0$, where, by definition, $x = x_0$ is a point at which $B_2^2\,(x)$, as given by Eq. (13), vanishes, i.e.,

$$(2 - m\mu_2)\,A^2(x_0) + (1-\mu_2)\,B_1^2(x_0) = 1. \tag{19}$$

At the point $x = x_0$, one must switch from Eqs. (14) and (15) to Eqs. (16) and (17), setting $B_2 \equiv 0$ at $x \geq x_0$. The continuity dictates to take the values $A(x \to x_0^-)$ and $B_1(x \to x_0^-)$ as the b.c. to Eqs. (16) and (17) at $x = x_0$, the b.c. at $x = +\infty$ being fixed by Eqs. (11).

The transition from Eqs. (14) and (15) to Eqs. (16) and (17) at $x = x_0$ provides for the continuity of all the functions $A(x)$ and $B_{1,2}(x)$, but their first derivatives suffer a jump at $x = x_0$. In fact, if the above-mentioned fourth-derivative term is added to Eq. (5), the jump of the derivative will be smoothed down in a narrow boundary layer. It should be stressed that the presence of the jump does not violate the applicability of the description in terms of the GL equations; the only problem is the absence of a detailed description of the narrow boundary layer in which the jump is smoothed by the fourth-order derivative term, but details of the inner structure of the narrow layer do not affect the global picture.

Thus, the transient layer between the two SC domains consists of two sublayers, described, respectively, by Eqs. (14) and (15), and by Eqs. (16) and (17), with the discontinuity of the derivative between them. The first (left) sublayer contains all the four spatial harmonics, hence is locally seems as a QP pattern. An important finding is that, in the case of small μ_2 considered here, the width of this sublayer is large, scaling $\sim \mu_2^{-1/2}$, according to Eqs. (14) and (15). The possibility to produce a broad QP stripe justifies all the consideration of the transient layer. The second (right) sublayer contains only three spatial harmonics, as $B_2 \equiv 0$ in it, hence it locally looks like a pattern quasiperiodic only in one direction [4]), and its width is ~ 1 (i.e., not specifically large), according to Eqs. (16) and (17).

This qualitative analysis of the transient layer's structure will be corroborated and illustrated by direct numerical results displayed in the next section. However, before

using the numerical methods, one can notice that the structure of the second (right) sublayer can be described in an approximate analytical form if we consider a special case when the value $A(x = x_0)$ at the internal boundary between the two sublayers is already close to the asymptotic value $A_{\rm lim}$, and, accordingly, the value B_1^2 is small (in other words, this is case when $0 < 1 - m \ll 1$). Then, we may set

$$A(x) \equiv A_{\rm lim} - a(x), \tag{20}$$

where $a(x)$ is positive but small and vanishes at $x = +\infty$. Using the smallness of $a(x)$ and B_1, one can simplify Eqs. (16) and (17):

$$\begin{aligned} (1/\sqrt{2})a'' - \left[\sqrt{2}(2-\mu_2)a - B_1^2\right] - \mu_1 B_1^2 &= 0, &(21)\\ B_1'' + \left[\sqrt{2}(2-\mu_2)a - B_1^2\right] B_1 - \frac{1}{2}\left(\mu_1 - 2\mu_2\right) B_1 &= 0. &(22) \end{aligned}$$

The b.c. for $a(x)$ at $x = x_0$ can be obtained from the expansion of the exact b.c. (19):

$$a(x_0) = \frac{B_1^2(x_0)}{\sqrt{2}\,(2-\mu_2)}\,. \tag{23}$$

The lowest-order approximate analytical solution to Eqs. (21) and (22), satisfying the necessary boundary conditions, is very simple:

$$B_1(x) = B_1(x_0)\,\exp(-\lambda x),\;\; a(x) = a(x_0)\,\exp(-2\lambda x), \tag{24}$$

with $\lambda = \sqrt{\left(\mu_2 - 2\mu_1\right)/2}$. Note that the condition necessary for λ to be real is again exactly tantamount to $m < 1$.

4 Numerical Results

To check the qualitative predictions for the structure of the transient layer obtained in the previous section, we performed a two-stage numerical integration of Eqs. (14) and (15), and then of Eqs. (16) and (17). To this end, the first pair of the equations was solved with the b.c. (12), continuing the solution until it hits a point where it satisfies Eq. (19) (recall the latter condition is equivalent to the vanishing of B_2^2). Actually, the numerical integration was performed, instead of the original coordinate x, in terms of a variable $\xi \equiv \tanh x$. This transformation is convenient because it maps the semi-infinite intervals $(\pm\infty, 0)$ of the variable x into finite ones $(\pm 1, 0)$.

The values $A(x_0)$ and $B_1(x_0)$, obtained from the solution of Eqs. (14) and (15), where then used to find a solution to Eqs. (16) and (17) in the interval $\tanh x_0 < \xi < 1$, satisfying the b.c. (11) at $\xi = 1$. In accord with what said above, a condition of the continuity of the first derivatives across the point $\xi = \tanh x_0$ was not imposed.

Three typical examples of the thus obtained numerical solutions are displayed vs. the coordinate x in Fig. 2, for three characteristic values $m = 0.75$, $m = 0.5$, and $m = 0.25$, the parameter μ_2, that must be small enough, being fixed in all the three cases as $\mu_2 = 0.1$. A characteristic feature clearly seen in all the cases is that, in accord with the prediction of the above analysis, a width of the left sublayer is essentially larger than that of the right one. It is also noteworthy that the solutions changes very little with a large change in m, i.e., the transient layer between the two SC domains is expected to be quite robust.

5 Conclusion

In this work, we have proposed an approach that makes it possible to the generate a two-dimensional quasiperiodic pattern in nonlinear dissipative systems where a direct generation of stable uniform quasiperiodic planforms is not possible. An eightfold pattern can be created in the form of a broad transient stripe between two domains filled by square cells, which are oriented under the angle of 45 degrees relative to each other. Using the symmetry of the configuration considered, the structure of the pattern was described in terms of a system of three coupled real stationary Ginzburg-Landau equations, which were analyzed by means of analytical and numerical methods. It was found that the transient quasiperiodic pattern exists exactly in a parametric region in which the uniform square-cell pattern is stable. Further, it was found that the transient layer consists of two different sublayers, with a derivative jump between them (that can be smoothed into an additional narrow boundary layer, if higher-order derivatives are added to the Ginzburg-Landau equations). The width of the sublayer that features the eightfold quasiperiodic pattern is found to be large, while the other sublayer (filled with a less interesting pattern, which is quasiperiodic only in one direction) has a width ~ 1. A broad stripe of a *twelvefold* QP pattern can be similarly generated as a transient layer between two domains of hexagonal cells oriented at the angle of 30 degrees.

It still remains to perform simulations of the full time-dependent Ginzburg-Landau equations, in order to directly test the dynamical stability of the broad transient layers. However, the numerically found robustness of the layers against the variation of the crucial control parameter m suggests that they have a good chance to be dynamically stable.

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Figure Captions

Fig. 1. The configuration giving rise to the transient layer filled with the eightfold quasi-periodic pattern between two domains of square cells oriented under the angle 45 degrees relative to each other..

Fig. 2. Numerically found structure of the transient layer corresponding to the configuration shown in Fig. 1. The small parameter μ_2 defined by Eq. (9) is fixed to be 0.1, while the control parameter m, defined by Eq. (10), takes values 0.75 (a), 0.50 (b), and 0.25 (c) (recall only the values $0 < m < 1$ make sense in the present context).

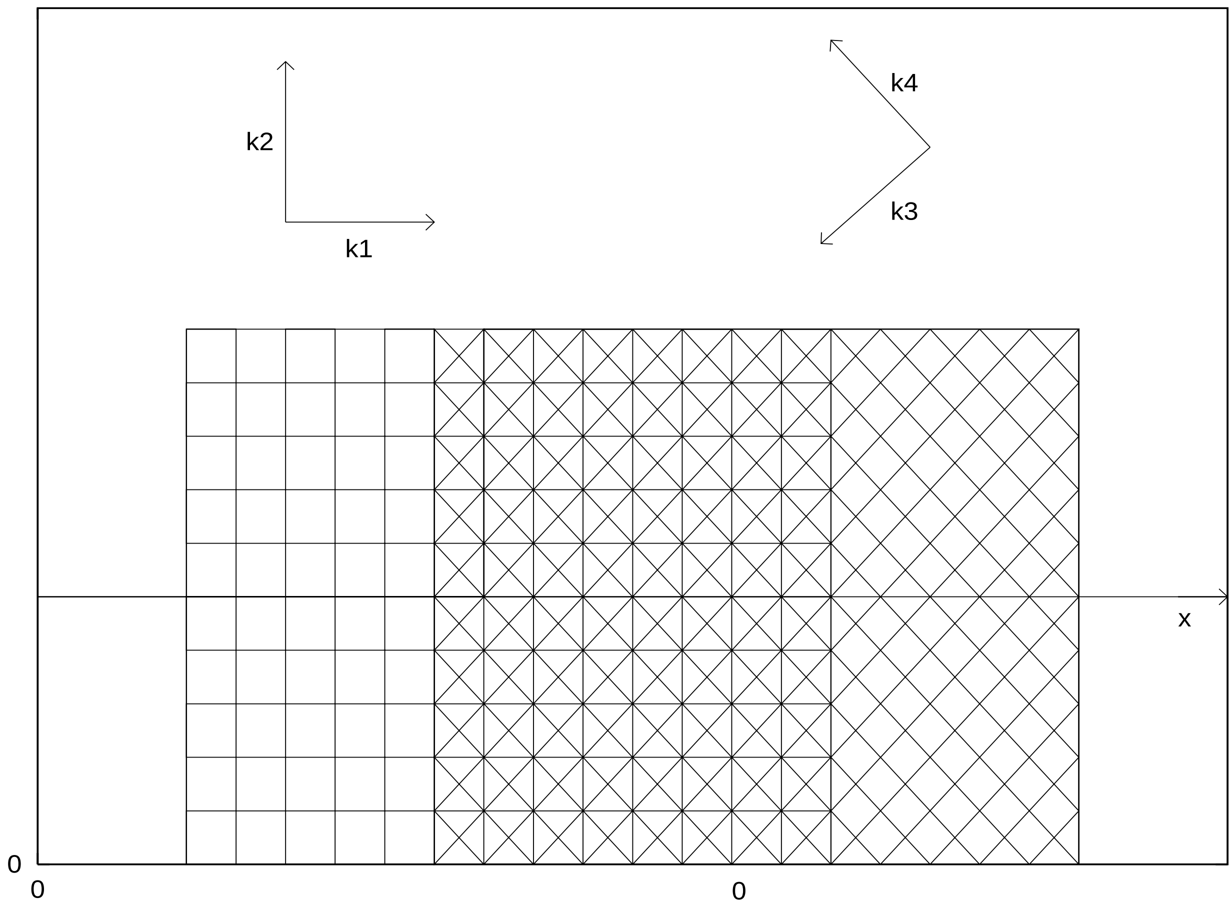
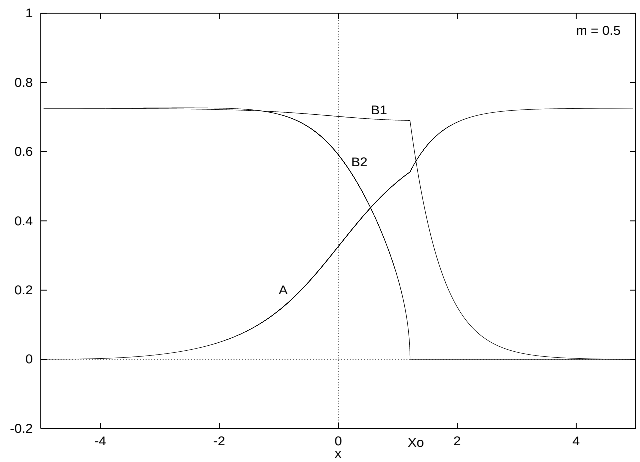


Figure 1:

(a)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B1
B2
A
Xo
x
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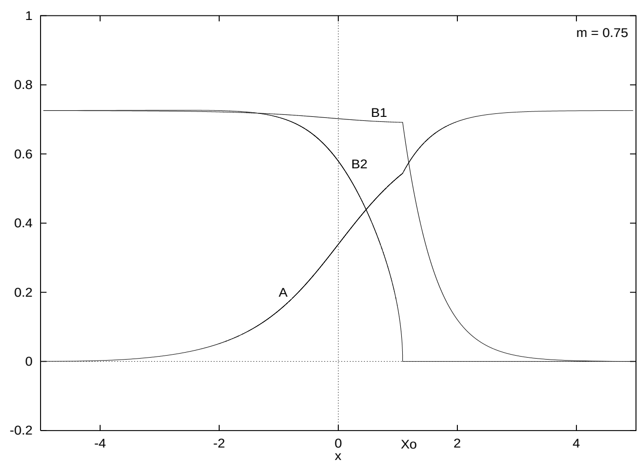
0.6
0.4
0.2
0
-0.2
-4
-2
0
2
4

(b)


m = 0.75
B1
B2
A
Xo
x
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0
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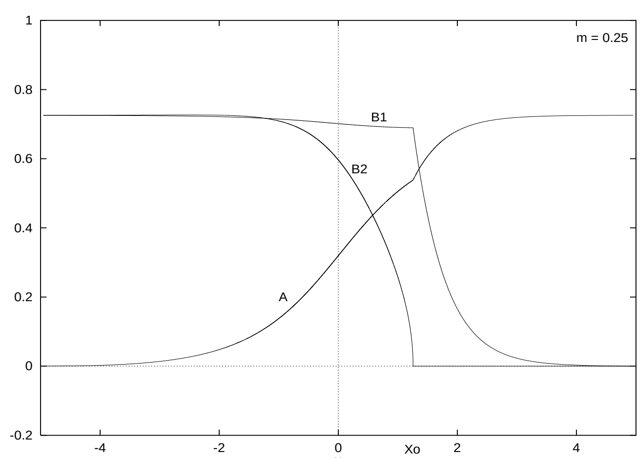
-4
-2
0
2
4

(c)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B2
A
Xo
x
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0.6
0.4
0.2
0
-0.2
-4
-2
0
2
4

Figure 2: